

Book Review

Making Sense of Math: How to Help Every Student Become a Mathematical Thinker and Problem Solver, by Seeley, Cathy L., Alexandria, Virginia, USA: ASCD, 2016

By

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This book deals with how students think and not only what they know. It also deals with how we teach as well as what we teach. This book helps us to think together about what it takes to become a mathematical thinker.

Who's Smart in Math?

Students have difficulty in math for many reasons. Some have learning problems or they may not speak English as their first language. Some believe in the myth that they are not naturally good in math. This is often reinforced by their parents and teachers.

There is not only one way to be good at math, but many ways. Some may be problem solvers. Others may be visual thinkers. Others may be thoughtful, slow processors of information. If we expand our ideas on this, many more students will see themselves as smart. Students can succeed if we change our thinking about what it means to be smart—not only in general, but also smart in mathematics.

Mindsets about Intelligence

We now know more about the nature of intelligence. In the book, *Mindset*, Carol Dweck (2006) talked about the differences between a fixed mindset regarding intelligence and a growth mindset. More evidence is supporting the growth mindset—that intelligence is far more changeable. Certain kinds of experiences can change intelligence.

Impact of a Student's Mindset

A student's mindset regarding his or her intelligence plays an important part in a student's willingness to challenge a problem. If students have a fixed mindset, they may think they are as smart as they ever will be, they may react negatively to hard problems. They may say, "I won't do it." With

a growth mindset students may be motivated to tackle difficult problems. We can help students develop a growth mindset.

Impact of a Teacher's Mindset

Teachers need to be careful not to have a low mindset for math students. They often can work through difficult problems if we give them the time. Mathematics gives them the opportunity to do this.

Implications for Teaching

We need to reconsider long-standing practices. We should think about how often, if ever, we use lecture, with the implication of having students listen to and remember what we have said. Lecture reinforces the idea that math is a set of rules. If we teach in this way, we confirm students' wrong ideas about math and themselves. They may think they are not math persons.

Another way of approaching this is a teacher-structured classroom instead of a teacher-centered classroom. This is where we encourage discussion around rich problems and mathematical ideas. This is sometimes called "upside-down teaching".

Seeing Below the Surface

There are many ways students may hide what they know or what talents they have. If they don't speak English well, this may be the case. Labels such as SPED, ED, ADHD, ELL and others may cause low expectations.

We must give the opportunity for limited English speaking students to share their mathematical thinking orally and in writing. One way is to have small group work so they are with proficient English readers. We need to challenge those who have a label or behavioral problems. It is important not only to focus on word problems or a rule they have just learned.

Math is Supposed to Make Sense

Many students and adults believe that math has to do more with abstract rules, memory tips and magic tips than making sense. The structure of many mathematical classrooms and the lecture approach can help a student to think that math doesn't make sense.

What it Means to Make Sense out of Mathematics

Making sense includes logic, reasoning, checking that the answer is reasonable and much more. When a student makes sense of what she's doing in math, she sees how an answer or a process follows from what came before and that the answer is reasonable and relates to what she knows in the past. There is an understanding that math makes sense as a discipline.

Mathematical Habits of the Mind

An important part of any effective mathematics program is the development of mathematical habits of the mind. These include: forming thought experiments, finding, articulating and explaining patterns, creating and using representations, generalizing from examples, articulating generality in precise language, and expecting mathematics to make sense. The one listed last is the most important.

When Students Don't Make Sense of Math What They Do

Some students think it doesn't matter if the answer makes sense or not. After all, this is math.

Expecting Math to Make Sense

Students and others often feel that math doesn't make sense. However, we can help students understand the inherent consistency in math and to understand numbers, operations, and relationships enough so that what they see and do in math makes sense to them.

It is important that students believe deeply that math makes sense in generating answers to problems, discussing their thinking and other students' thinking and learning new material. If something does not make sense, it should cause them to be concerned. That should cause them to take action until it does make sense. If another student's answer does not make sense, it should cause others to ask clarifying questions. If they need clarification from the teacher, they should ask questions. Every student and teacher should believe that math is supposed to make sense.

What Mathematics Do Students Need?

The Common Core Standards for Mathematics are based on concepts, computation and problem solving. However, some programs emphasize problem solving too much.

Mathematical Building Blocks: Concepts, Computation, Problem Solving.

In order for students to learn mathematics in useful ways they need to make sense of what they are learning (concepts), and know some skills and computational procedures (computation), and use what they are learning to solve problems (problem solving). Of course, computation is important, but number sense and operational sense are also very important. The most important thing students need to know is the ability to use the concepts and skills they learn to solve problems. They need to have confidence to tackle problems that may not look like problems they have seen before. An example would be determining what combinations of vehicles (cars, vans and busses) might accommodate a group of students on a field trip.

Fluency, Speed, and Calculators

Students with a strong understanding of what an operation means and a strong sense of the numbers involved are far more likely to remember a fact like 4×7 . Making pictures of four groups of seven can help students with this. Fluency means that students can access the facts they need when they need them. Calculators are important and can be powerful to enhance, not replace, math teaching and learning. Calculators should not be used in certain situations, such as when developing mental math skills or key operational procedures. Calculators can offer students the potential to work on complex problems. Students often use technology in situations in which it may not be helpful or appropriate.

Can Struggling Be Good for Students?

Productive struggling can help students learn math as long as we provide the structure and support for productive struggle. Productive struggle is one of eight effective practices for teaching identified by the National Council of Teachers of Mathematics.

How Do We Withhold Struggling?

Unfortunately, productive struggling is often withheld in classrooms. It may be withheld for those we see as less able or for other reasons.

Sometimes we emphasize rules too much and don't let students productively struggle. We may also do this when we are rushing through material to cover objectives. We might rush advanced students through material too quickly. If we show students how to solve a particular problem, we might take away the chance for them to think about it. The way we ask and answer questions may end productive struggling if we give too many directions. In Japan, teachers tend to use problems to learn the intended mathematics, instead of seeing the solution as the end result.

Embrace Mistakes

It is important to help students look at mistakes as they go through the process. Teaching with mistakes is an important part of math learning. Mistakes are an important part of learning. If we embrace mistakes, students will feel free to take a risk. We might say mistakes are expected, inspected and respected.

The Wrong Kind of Struggle

It does not make sense if it does not lead to making sense of math. Struggle is not helpful if students become bogged down. If they are struggling unproductively, it is important for them to understand why. For example, we may ask them the reason they see a problem in a certain way. Some students may need only a small clarification. Some students may be only one question away from productive struggle.

Opportunities for the Right Kind of Struggle

Students need three particular kinds of opportunities: productive struggle to help them strengthen their thinking skills; deliberate practice to gain confidence; and explicit connections for seeing how new ideas, concepts, and skills connect to what they know already. By these we can help students become proficient mathematical thinkers.

What if We Turn Teaching Upside Down?

To help students we need to create classrooms that focus on thinking and sense making. We need to have rich, worthwhile tasks. This may call for a shift in how we teach.

An Environment for Making Sense

We should create classes where students realize that mathematical discourse is the way a math class should be. They need to see classrooms where they can tackle problems, talk about them and respectfully argue with each other. This needs to become part of the “math culture”.

Mistakes as a Stimulus for Discussion

When students process a mistake and arrive at a correct answer, something positive happens. If we don't emphasize getting correct answers quickly it may allow us to teach differently.

Focus on Learning

We can help students learn math better if we shift our goal to using problems as a way to learn mathematical content. This will help students dig into math ideas and concepts when they work through problems. An example of this for young children would be a problem where there are 10 cars in a parking lot. Some are red and some are black. How many combinations of cars are there? The focus on this problem is building number sense.

Start with a problem

By creating a safe and fertile classroom environment we can imagine problems can be a powerful tool in learning math. It may be helpful to give students an engaging problem and have them explore it. It is best to do this with a partner or, preferably, in a small group. We learn from what works well or what doesn't work. We must help students know what mathematics they have learned.

The Problems We Choose

In our teaching choosing appropriate tasks becomes the essential issue. It is important to have tasks that promote reasoning and problem solving. We need to select opportunities that afford for engagement, reasoning and discussion. A good problem may have more than one path toward a solution. We may ask them how to find $\frac{1}{6}$ of $\frac{1}{4}$ of a shape by using pattern blocks. We should show students that math can be used outside of school.

Move to You–We–I

I am going to present you (students) the procedure or concept for this lesson.

We (with the help of the teacher) are going to practice together until we can do it on our own.

You (students) will do the procedure and apply the procedure or concept for solving related word problems.

This type of teaching is sometimes called gradual release of responsibility.

We need to move to a You–We–I model.

You (students) are going to mess around with a problem.

We (students and teacher) need to have a discussion about what has been tried, what was found, what worked, what didn't, and why certain approaches may be productive or not.

I (the teacher) will help you connect your ideas to mathematical ideas and procedures.

Discover New Stars

With this student-centered, problem-focused classroom, new stars may evolve. Some may find they are really “math people”.

Communicate with Parents

We need to reach out to families and caregivers to let them know that we may be doing things somewhat differently. By implementing researched-based practices students may be able to tackle more challenging problems.

What about the Test?

If we cover content without paying attention to learning it can have disastrous results in terms of breadth and depth of knowledge. If they remember quickly covered facts or skills for a test, that learning may be short-lived. We can help students learn mathematics deeply and also test well.

With highest priority topics, concepts and skills we can spend more time on in-depth problems. By doing so, the students learn in lasting ways that can be assessed. Many tests are being developed with in-depth assessment tasks. We need to make sure students have experience in solving a wide-range of problems. In well thought out education, students will learn to deal with problems and be prepared for tests.

Where Can I Find Support?

It helps if we can go through our process of changing our teaching with at least one colleague. It is even better if teachers in a grade level or course, or even a whole school work together. We can gain support by working with others informally and formally.

Work Within and Across Grades and Courses

If we specify a small number of courses for each grade, it will be helpful. This way we can spend time on topics with the highest priorities. If we agree on a few top priorities, we can use valuable instructional time to allow students to at least finish the most critical topics and skills. This type of planning can be effective in working with colleagues in adjacent grades. If there can be continuity across schools, this can be very helpful.

Rely on Your Professional Learning Community

To develop a professional learning community, it is important to think of the meaning of the words.

Professional: Centered on a professional commitment to help students learn important content, processes and thinking skills.

Learning: Have the attention of learning to include the critical elements of making sense of mathematics and learning to think mathematically.

Community: Make sure we take advantage of the power of our collaboration to learn from each other.

Conclusion

We need to change how mathematics looks. We must help students have more knowledge and be flexible thinkers who can approach problems from different perspective and apply what they know in unexpected situations. We need help develop creative thinkers and collaborative problems solvers.

Critique

Cathy L. Seeley is an excellent writer about mathematics and has experience as a teacher, district mathematics coordinator, and Texas state mathematics director for grades K–12. She recently retired as Senior Fellow with the Charles A. Dana Center at the University of Texas.

She writes in a very clear and easy to understand way, yet covers new ideas in the teaching of mathematics well.

The book is short (only 44 pages) so it would be helpful to have more examples. It gives the reader new things to try in the teaching of mathematics.